Neutron-halo structure of light nuclei studied with effects of deformations and orientations included

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Neutron-halo structure of light nuclei studied with effects of deformations and orientations included

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Abstract

Based on the cluster-core model, the effects of nuclear deformations and orientations on the halo structure of the observed and proposed cases of neutron-halo nuclei is analyzed. The calculations are performed for the deformed structures using quadrupole ($\beta_2$) deformations with ‘optimum’ orientations as well as including higher multipole deformations ($\beta_2$-$\beta_4$) with the ‘compact’ (hot) orientations. The interesting result is that, although potential energy surfaces are modified with the inclusion of deformation effects up to $\beta_4$, but the 1n- and 2n-halo nature remains intact for almost all cases investigated here, except for $^{17}\text{B}$ and $^{22}\text{C}$ nuclei. However, in some cases ($^{22}\text{O}$, $^{26}\text{F}$, $^{27}\text{F}$), the choice of higher multipole deformations up to hexadecapole is shown to be rather sensitive. In addition, the relevance of the use of different nuclear proximity potentials is also explored in the context of the halo nature of neutron-rich light nuclei. The possible role of $Q$-value and angular momentum effects are also addressed.

Keywords: deformations and orientations, nuclear structure, cluster core model

1. Introduction

Nuclei far from the stability line have presented an extensive general view of nuclear structure, inaccessible from stable nuclei. Since the introduction of shell model for explaining the structure of stable nuclei and the, so-called, magic numbers, much progress has been made with the availability of radioactive nuclear beams for understanding the evolution of nuclear structure away from the valley of stability. The size and distribution of matter have played a central role in understanding the general features of a nucleus. For stable nuclei, the neutron and proton distributions exhibit essentially identical radii, except for some light nuclei with
large neutron excess and very weak binding, that large differences have been found. In other words, these light nuclei exhibit a high probability of the presence of one or two loosely bound neutrons at a large distance from the rest of nucleons. Such ‘halo’ systems are well described by a core, resembling a normal nucleus, surrounded by an extended valence neutron density distribution [1]. All halo nuclei have the same features of having a large nuclear matter radius (far beyond the nuclear-interaction range) and low binding energies of valence nucleons surrounding the core. Based on these characteristics, the structure properties of highly neutron-rich halo nuclei (see, for example, the review [2]) lying near the neutron drip line has attracted the interests of many scientists and researchers all over the world. The existence of proton halos has also been observed [3, 4] in some proton-rich nuclei. However, relatively speaking, the presence of repulsive Coulomb interaction hinders the formation of proton halos.

The neutron-halo nuclei have so far been limited to only the light nuclear systems, like $^{11}$Be, $^{19}$C (with one-neutron halos), and $^{6,8}$He, $^{11}$Li, $^{14}$Be, $^{17}$B (with two-neutrons halos). The 1n halo nucleus $^{31}$Ne [5] and the 2n halo nucleus $^{22}$C [6], heavier than $^{19}$C, have been identified very recently. In addition to the above mentioned cases where direct experimental information is available, many more cases are proposed as likely candidates, listed in table 1, respectively.

<table>
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<tr>
<th>Structure</th>
<th>Nucleus</th>
<th>$S_{1n}$ (KeV)</th>
<th>$S_{2n}$ (KeV)</th>
<th>Spherical/Deformed</th>
<th>Cluster</th>
<th>Core</th>
<th>Possible magic number</th>
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<td>10/11</td>
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<td>2015.4</td>
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$^a$ $\beta_2$ deformed case.
$^b$ $\beta_2$-$\beta_4$ deformed case.
for the 1n and 2n-halo nuclei. The main source of information for the halo structure of a nucleus is the one/two neutron separation energy. These nuclei exhibit particularly low one-neutron separation energy \( S_1 = B(Z, N) - B(Z, N-1) \) or two-neutron separation energy \( S_{2n} = B(Z, N) - B(Z, N-2) \) in comparison with the mean binding energy of the stable nuclei, which lies around 8 MeV/nucleon. It should be noted that whereas one-neutron halo nuclei exhibit \( S_1 < S_{2n} \), the two-neutron halo nuclei have \( S_{2n} < S_1 \). In fact, the two-neutron halo nuclei are ‘borromean’, i.e., these nuclei exhibit a three cluster structure in which none of the binary sub-systems are bound; for example \(^{11}\text{Li}\) is bound through neither \(^{10}\text{Li}\) nor through di-neutron, since the existence of a bound di-neutron is not yet observed \([7, 8]\) in the ground state. Therefore, 1n and 2n halo nuclei are investigated, respectively, via two-body (n+core) and three-body (n+n+core) structures.

Although the information on halo structures for some of the nuclei is obtained experimentally via Coulomb breakup or nuclear dissociation mechanism \([9, 10]\), an appropriate theoretical treatment is a must for an overall understanding of any concept and hence it is worth performing a detailed theoretical analysis in reference to halo structures of light nuclei. In view of this, the halo structures of a variety of neutron (and proton) drip-line nuclei have been investigated by Gupta and collaborators \([11-13]\) in terms of their, so-called, cluster-core model (CCM) based on the core+valence nucleons picture. It was further shown \([14]\) that the halo structure is associated with the closed shell effects of the core nucleus. Since the shape of a nucleus provides an intuitive understanding of the spatial density distributions, therefore, together with shell effects, it is expected that nuclear deformations and orientations should play an important role in context of halo nuclei. In order to look for such effects, we have further explored in this paper, the above study \([12, 13]\) for neutron-halo nuclei, using the (neutron) CCM for analyzing the influence of nuclear deformations and orientations on the halo structure (fragmentation path) of neutron-rich halo nuclei.

The CCM finds its basis in the well known quantum mechanical fragmentation theory where the halo nature of a possible neutron drip-line nucleus is studied via the minima in fragmentation potential energy surfaces (PES), which in turn corresponds to the most probable halo-configuration. This method works equally well for proton-rich nuclei, provided the Coulomb repulsion of protons in proton-clusters is added. In CCM, not only the shapes of halo nuclei are important, but also of all other possible fragments that a halo nucleus could be made-up of, since the fragmentation process depends on the collective clusterization approach. It is relevant to mention here that all the 1n-halo nuclei \((^{11}\text{Be}, ^{14}\text{B}, ^{15}\text{C}, ^{17}\text{C}, ^{19}\text{C}, ^{22}\text{N}, ^{22}\text{O}, ^{24}\text{O}, ^{24}\text{F}, ^{29}\text{Fe}, ^{31}\text{Ne})\) and 2n-halo nuclei \((^{6}\text{He}, ^{8}\text{He}, ^{11}\text{Li}, ^{12}\text{Be}, ^{14}\text{Be}, ^{17}\text{B}, ^{19}\text{B}, ^{22}\text{C}, ^{23}\text{N}, ^{27}\text{F} \) and \(^{29}\text{F}\)) considered here are deformed, and hence the deformation effects seem indispensable which were not analyzed explicitly in the earlier works \([12, 13]\) on CCM. Instead, knowing that the radii of halo nuclei change considerably from nucleus to nucleus and from one isotope to another isotope, in the earlier calculations \([12, 13]\), the authors used the charge- and mass-dependent equivalent spherical radius \( R_0(Z, A_i) \) in \( R_i = R_0 A_i^{1/3} \), taking (or interpolating) it’s values for \( Z \leq 10 \) from the experimental \([15, 16]\) or theoretical \([17]\) data and for \( Z > 10 \) from the theoretical estimates of \([18]\). This means to have in fact included the surface/ deformation effects indirectly, though its contribution to halo structure of nuclei not assessed explicitly. Alternatively, the matter radius of a neutron-rich nucleus could be calculated on the CCM picture itself by using a prescription that relates its square to the square of core radius and a (squared) radius obtained as the expectation value of \( r^2 \) computed with the wave function of the last neutron(s), assumed to be a single-particle neutron radial wave function (for details, see \([19, 20]\)).

The aim of the present work is to investigate the effects of nuclear deformations explicitly, by accounting for the quadrupole as well as higher multipole deformations, like the octupole
and hexadecapole, on the behavior of PES in both 1n- and 2n-halo nuclei. For $\beta_2$ deformations, the ‘optimum’ orientations $\theta_{\text{opt}}^i$ are used [21], whereas for higher multipole deformations up to hexadecapole ($\beta_2 - \beta_4$) the ‘compact’ orientations $\theta^i$ of the constituting fragments are taken in to account [22]. The possibility of identifying new magic numbers is also explored in these exotic nuclei. In addition, the halo structure via fragmentation path is also investigated in reference to the use of different proximity potentials within the CCM approach. In other words, the comparative importance of using the nuclear proximity potentials (Prox 1977, Prox 1988, and Prox 2000) having different isospin and asymmetry dependant parameters is checked in context of halo structure of neutron-rich light nuclei.

The paper is organized as follows. Section 2 gives a brief account of the CCM with effects of deformations and orientations of nuclei included. Various nuclear proximity potentials are also introduced in this section. Section 3 presents the calculations for both 1n- and 2n-halo nuclei. Finally, the results are summarized in section 4.

2. Methodology

The potential energy for a cluster-core configuration ($A_2, A_1$) of a nucleus $A$ in CCM is defined as the sum of binding energies, the Coulomb repulsive interaction, the additional attraction due to nuclear proximity force and the centrifugal potential due to angular momentum $\ell$:

$$V(A_1, A_2, R, \ell) = -\sum_{i=1}^{2} B(A_i, Z_i) + V_C(R, Z_i, \beta_{\lambda i}, \theta_i)$$

$$+ V_P(R, A_i, \beta_{\lambda i}, \theta_i) + V_\ell(R, A_i, \beta_{\lambda i}, \theta_i),$$

(1)

$B_i$ $(i = 1, 2)$ are the binding energies of the two fragments, taken from experimental data of Audi–Wapstra [23] and hence the microscopic shell effects are included in equation (1). Wherever the experimental $B$’s are not available, the theoretical binding energies of Möller et al [24] are used. $V_C$, $V_P$ and $V_\ell$ are, respectively, the Coulomb, nuclear proximity and angular-momentum-dependent potentials between two deformed and oriented nuclei, whose details can be seen, e.g., in [25, 26]. The $\ell$-dependent potential is given by

$$V_\ell = \frac{\hbar^2 \ell(\ell + 1)}{2I_5}$$

(2)

with $I_5 = \mu R^2 + \frac{2}{3} A_1 m R_1^2(\alpha_1) + \frac{2}{5} A_2 m R_2^2(\alpha_2)$, the moment-of-inertia in sticking limit. The details of nuclear proximity potential $V_P$ are discussed below.

In the above equations, $\theta_i$ $(i = 1, 2)$ is the orientation angle between the nuclear symmetry axis and the collision $Z$-axis, measured in the anticlockwise direction (see, e.g., figure 1 of [21]). The deformation parameters $\beta_{\lambda i}$ $(\lambda = 2, 3, 4)$ of the nuclei are taken from [24], and the orientations $\theta_i$ are the ‘optimum’ orientations for up to $\lambda = 2$ from [21] and the ‘compact’ orientations, obtained as per prescription in [22], for the case of $\lambda > 2$. Note that the neutron-clusters are taken to be spherical, with the orientation of the corresponding core depending on its $\beta_2$ or $\beta_4$ value. The ‘optimum’ orientations $\theta_{\text{opt}}^i$ are uniquely fixed on the basis of the signs (+, −, or spherical) of quadrupole deformations $\beta_2$ of nuclei alone [21], which also holds good for small positive or any negative value of the hexadecapole deformations $\beta_4$ [22]. In order to address large positive $\beta_4$, i.e., $\beta_4 \gg 0$ for, say, prolate deformed nuclei [22], $\theta_{\text{opt}}^i$ are in fact the ‘compact’ orientations $\theta^i$ which refer to collisions taking place at the minimum (smallest) interaction radius, having the highest interaction barrier; and so on for the oblate deformations of nuclei [22].

The collective fragmentation potential $V(A_1, A_2, R, \ell)$ brings out the fragmentation structure of a nucleus. Since the shape of the potential is known to be nearly independent
of the choice of value of $R$ [27], and in view of our dealing here with light nuclei, we consider only the touching configuration, i.e., $R = R_1 + R_2 = R_t$, with $R_t$ defined as

$$R_i(\alpha_i) = R_0_i \left[ 1 + \sum_\lambda \beta_i(0) Y_\lambda(\alpha_i) \right]$$  (3)

where, $R_0_i = 1.28A_i^{1/3} - 0.76 + 0.84_i^{1/3}$. In equation (3), $\alpha_i$ is an angle that the nuclear symmetry axis makes with the radius vector, measured in the clockwise direction (see, e.g., figure 1 in [28]). The charges $Z_i$ in equation (1) are fixed by minimizing the sum of the two binding energies in $Z_1$ (or $Z_2$). The binding energy for a cluster with $x$ neutrons ($x \geq 1$) is taken to be $x$ times that of the one-neutron binding energy (equivalently, mass excess $\Delta m_n$), i.e., $B(A_i = xn) = x \Delta m_n$, which means that nucleons in these clusters are taken to be unbound, as suggested theoretically [29, 30] and in experiments [7, 8].

The nuclear proximity potential in equation (1), for deformed, oriented nuclei, is given by [28],

$$V_p(s_0) = 4\pi R b \Phi(s_0)$$  (4)

where $b = 0.99$ is the nuclear surface thickness and $R$ is the mean curvature radius (for details, see [28]). $\Phi$ in equation (4) is the universal function, independent of the shapes of nuclei or the geometry of the nuclear system, but depends on the minimum separation distance $s_0$. Various versions of nuclear proximity potentials used in the present work are as follows.

2.1. Proximity 1977 (Prox 77)

For this proximity potential, the universal function is given by Blocki et al [31],

$$\Phi(s_0) = \begin{cases} -\frac{1}{4}(s_0 - 2.54)^2 - 0.0852(s_0 - 2.54)^3 & \text{for } 0 < s_0 \leq 2.54, \\ -3.437 \exp \left( \frac{5.75}{s_0 - 0.75} \right) & \text{for } s_0 \geq 2.54, \end{cases}$$  (5)

respectively, for $s_0 \leq 1.2511$ and $s_0 \geq 1.2511$.

The surface energy constant used for this potential is

$$\gamma = \gamma_0 \left[ 1 - k_s \left( \frac{N - Z}{A} \right)^2 \right] \text{MeV fm}^{-2}.$$  (6)

Here, $N$ and $Z$ are the total neutrons and protons numbers, and the coefficients $\gamma_0$ and $k_s$ were taken to be 0.9517 MeV fm$^{-2}$ and 1.7826, respectively. In this work, we use this functional form for $\Phi$, except otherwise stated.

2.2. Proximity 1988 (Prox 88)

Reisdorf [32] used a modified version of proximity potential, labeled ‘Proximity 1988’ or ‘Prox 88’, where in he took the value of coefficients $\gamma_0$ and $k_s$ to be 1.2496 MeV fm$^{-2}$ and 2.3, respectively, from the improved mass formula of Möller and Nix [33]. The universal function is kept the same as for Prox 77. It is to be noted that this set of coefficients gives stronger attraction, i.e., deeper pocket, and includes stronger isospin effect, as compared to Prox 77.

2.3. Proximity 2000 (Prox 00)

The universal function in this version of proximity potential is taken from Myers and Swiatecki [34], as

$$\Phi(s_0) = \begin{cases} -0.1353 + \sum_{n=0}^{5} c_n(s_0/n + 1)(2.5 - s_0)^{n+1} & \text{for } 0 < s_0 \leq 2.5, \\ -0.0955 \exp[(2.75 - s_0)/0.7176] & \text{for } s_0 \geq 2.5, \end{cases}$$  (7)
where $s_0 = R - R_1 - R_2$. The values of different constants $c_n$ are $c_0 = -0.1886$, $c_1 = -0.2628$, $c_2 = -0.15216$, $c_3 = -0.04562$, $c_4 = 0.069136$, and $c_5 = -0.011454$. For further details of surface energy coefficient and nuclear charge radius, etc., see [34].

Note that Prox 00 uses a completely different set of parameters in terms of universal function, surface energy constant and nuclear charge radius, as compared to Prox 77 and Prox 88 potentials which are the same, except for two different value of coefficients in surface energy term. Therefore, the behavior of Prox 00 is expected to be different from the other two potentials Prox 77 and Prox 88.

3. Calculations and results

In this section, we study the characteristics of the fragmentation path for a large number of neutron-rich nuclei using the CCM, covering almost all the known 1n- and 2n-halo nuclei, listed in table 1 (24 cases in total). The cluster-core configurations, corresponding to minima in these PES, represent the most probable configurations, i.e., they occur with relatively larger preformation probabilities, compared to their neighbors. Of these cluster-core configurations, we concentrate here only on the one where a cluster of neutrons is involved. Such a cluster will behave like a neutron halo and provide a loosely bound configuration to core.

Table 1 lists all the cases of established halo nuclei, as well as some other nuclei where a halo structure is expected, along with their 1n and 2n separation energies, $S_{1n}$ and $S_{2n}$, calculated by using the binding energy table of [23] and, wherever not available, the theoretical values of [24] are used. First of all, we look at the behavior of $V(\eta)$ (or $V(A_{2})$) for the light fragment $A_{2}$ alone, illustrated in figure 1 at $\ell = 0$ for (a) $^{11}$Be and (b) $^{6}$He which are, respectively the cases of 1n- and 2n-halo nuclei. The calculations are done for spherical as well as quadrupole ($\beta_2$) deformed fragments having ‘optimum’ orientations $\theta_i^{\text{opt}}$, taken from table 1 of [21]. Note that the orientations are ‘optimized’ (uniquely fixed) on the basis of $\beta_2$ alone which manifests in the form of ‘hot (compact)’ and ‘cold (non-compact)’ configuration. However, for investigating the role of higher-order deformations ‘compact’ orientations [22] must be used instead of
Figure 2. The (fragmentation) potential energy at $\ell = 0$, for (a) 1n- and (b) 2n-halo clusters formed in various halo nuclei, taking the two fragments as spheres, and with $\beta_2$ alone having ‘optimum’ orientations.

‘optimum’ orientations [21]. It is evident from figure 1 that although PES are modified, but the deepest minima occurs clearly at 1n + core configuration for $^{11}$Be and 2n + core configuration for $^6$He, irrespective of the use of spherical or deformed $\beta_2$ fragmentation. Furthermore, we have carried out comparison of ‘hot (compact)’ and ‘cold (non-compact)’ orientations on the respective halo structure of $^{11}$Be and $^6$He nuclei. The ‘hot compact’ configuration corresponds to smallest interaction radius and highest barrier whereas the ‘cold non-compact’ configuration corresponds to largest interaction radius and lowest barrier. Our analysis of figure 1 clearly shows that the fragmentation path is more favorable (minimum potential energy) for ‘hot compact’ in comparison to ‘cold non-compact’ configuration for cases of both 1n- and 2n-halo nuclei. In reference to the above observation, in this paper, we concentrate only on the ‘optimum’ orientations of ‘compact hot’ configuration.

To investigate the possible role of deformations further, figures 2(a) and (b), respectively, show the result of our CCM calculations where the potential energy corresponding to 1n- and 2n-clusters is plotted as a function of the halo-nucleus mass number $A$. One can clearly see from figure 2 that the choice of either spherical or deformation effects up to $\beta_2$ alone in the fragmentation potential of 1n-clustering in various halo nuclei do not play any significant role, i.e., they do not influence the status of 1n-minima in $V(A_2)$. On the other hand, the inclusion of $\beta_2$-deformation and ‘optimum’ orientations for 2n-halo nuclei shifts the minima in PES to 1n and 4n, respectively, for $^{17}$B and $^{22}$C parent nuclei, which have been discussed in figure 3 below. This signifies that deformation and orientation effects up to quadrupole ($\beta_2$) are more sensitive for the case of 2n-halo systems. In addition, we further observe in figure 2 that the relative emergence of 1n-halo clusters (relative minima) is more prominent for $^{15}$C and $^{24}$F light nuclei which shifts to $^6$He, $^{15}$Be and $^{27}$F for the 2n-halo clusters.

Figure 3 shows the result of our calculation for the above mentioned two cases of $^{17}$B and $^{22}$C 2n-halo nuclei. It is relevant to mention here that the minima in a PES arise mainly due to the shell effects of either one or both the fragments [27]. Therefore, it is of further interest to identify the new magic numbers and molecular structures on the basis of PES calculated within the CCM. For the angular momentum part of the potential, we have considered the cases of $\ell = 0$ and $\ell = 1, 2, 3 \hbar$, chosen arbitrarily. Both the position and depth of potential energy minima in $V(A_2)$, in general, are found to be independent [35] of the contribution of $\ell$-dependent term in it. The PES in cases of $^{17}$B and $^{22}$C nuclei show that the 2n + core configuration is clearly prominent for spherical choice of fragmentation. The proton
and neutron number corresponding to the most preferred core configuration are listed in table 1, along with the calculated $S_{1n}$ (or $S_{2n}$) energies. We note that, with the inclusion of $\beta_2$ deformation and ‘optimum’ orientations at $\ell = 0$, the minimum shifts to $1n +$ core configuration for $^{17}$B nucleus and to $4n +$ core configuration for $^{22}$C nucleus. However, for $^{17}$B, the lowest neutron separation energy is for $2n$ separation, $S_{2n}$, as listed in table 1. On the other hand, for $^{22}$C, $S_{3n}$ comes out to be 5606.7 KeV ($\gg$ 1 MeV) and thus $S_{2n} < S_{3n}$. Thus, the neutron separation energy hypothesis characterize both $^{17}$B and $^{22}$C as the $2n$-halo structures. Interestingly, we find in figure 3(b) that the most probable cluster-configuration clearly occurs for $1n$ at $\ell = 0$ and $\ell = 1\hbar$ which gradually changes to $2n$ configuration with increase in $\ell$. 

Figure 3. Fragmentation potential for the decay of $^{17}$B and $^{22}$C nuclei, plotted at different $\ell$ values, for (a, c) spherical, and (b, d) $\beta_2$-deformed choice of nuclei.
Figure 4. Same as for figure 3, but for $^{29}$F halo nucleus.

value, possibly due to the effect of deformation in the moment of inertia through $V_\ell$ (refer, equation (2)). This implies that angular momentum $\ell$ effect, in addition to deformations and orientations, is important in reference to the structure of $^{17}$B halo nucleus. On the other hand, for $^{22}$C nucleus, 4n configuration is preferred irrespective of $\ell$ contribution added or not added. Although amongst the 1n to 3n clusters, 2n decay seems relatively more probable for spherical nuclei, and hence this switching of halo status from 2n to 4n with the inclusion of deformations provide an interesting case of investigation in this mass region.

Furthermore, the study of $^{29}$F nucleus in figure 4 is of special interest which clearly gives 2n-halo for the choice of $\beta_2$, though in the PES, for both $\ell = 0$ and $\ell > 0$ cases, the 2n + core minima is almost as deep as 4n + core configuration for the fragments taken as spheres. Also the comparison of figures 4(a) and (b) clearly shows that the structure of PES changes significantly as one goes from spherical to deformed choice of fragmentation at all $\ell$-values. This behavior is observed to be significantly different as compared to figures 1 and 3, and earlier studies [12–14], where the general behavior of PES shows an increasing trend with fragment mass $A_2$. However, in the present case, it seems that deformations and orientations of decaying fragments increase the probable emergence of the very heavy fragments (deeper minima for fragments heavier than mass 9). Thus, $^{29}$F provides an interesting case to investigate the 2n + core or 4n + core configuration, and hence the halo structure of this nucleus. It is relevant to mention here that $S_{2n} (=875.7$ KeV) is lower than $S_{4n} (=2501.1$ KeV), therefore $^{29}$F is a 2n-halo nucleus from the point of view of the separation energy.

Next, in order to see the relevance of higher multipole deformations (up to hexadecapole $\beta_4$) on the structure of halo systems, the fragmentation behavior is studied explicitly for $^{22}$O, $^{23}$O, $^{24}$O, $^{26}$F, $^{28}$F, $^{29}$Ne, and $^{31}$Ne cases of 1n-halo nuclei and $^{23}$F and $^{25}$F cases of 2n-halo nuclei, considered here in table 1. The choice of these nuclei depends on the availability of estimated $\beta_4$ values [24] for their cluster-core products. Despite the change in PES, we observe that, with the inclusion of $\beta_2$-$\beta_4$ deformations, 1n- and 2n-halo structures remain the same in all cases, except for $^{22}$O, $^{26}$F and $^{28}$F nuclei. The clear preference for the 1n-cluster in $^{23}$O and $^{26}$F nuclei at $\ell = 0$ is evident from figures 5(a) and (b), irrespective of the use of spherical or $\beta_4$ configuration, which shifts to 2n-cluster for the choice of deformations included up to hexadecapole ($\beta_4$). Similarly, for the 2n-halo $^{27}$F nucleus, the PES in figure 5(c) show deepest minima at 1n + core for the use of $\beta_4$ and ‘compact’ orientations. In our analysis of figure 5(a),
we find that $^4\text{Li} + \text{core}$ configuration is also one of the probable configuration in $^{22}\text{O}$ for the case of $\beta_2$-deformed choice of fragments. However, the most preferred cluster-core configuration for $^{22}\text{O}$ is $1\text{n}+^{21}\text{O}$. It is further observed that the fragments corresponding to minima in PES do not remain same in going from spherical to deformed ($\beta_2$-alone or $\beta_2-\beta_4$), specifically for $^{26}\text{F}$ and $^{27}\text{F}$. This signifies the fact that fragmentation path of halo nuclei is significantly influenced by the inclusion of deformation and orientation effects.

Table 1 clearly shows that the cases of both 1n- and 2n-halo structures are identified on the basis of the CCM calculations (PES minima). However these predictions differ in few cases in comparison to the other known hypothesis of separation energy. The notable exceptions are the two cases of 2n-halo nuclei, $^{12}\text{Be}$ and $^{23}\text{N}$, where $S_{1n}$-value is less than $S_{2n}$. Also, the neutron separation $S_{1n}$ for $^{22}\text{O}$, $^{23}\text{O}$, $^{24}\text{O}$, $^{24}\text{F}$, $^{29}\text{Ne}$ and $S_{2n}$ for $^8\text{He}$, $^{12}\text{Be}$, $^{23}\text{N}$ nuclei are found to be much larger than $\sim 1$ MeV. It is clearly evident from table 1 that the PES analysis of all the n-halo nuclei allows us to predict new magic numbers at $N = 6, 12, 14, 16$, and $18$ and $Z = 6$, in addition to the well known $N = Z = 2, 8$, and $20$. These magic numbers are shown to be associated with the closed shell effects of the core nucleus. Note that these observations are made on the basis of fragmentation potential used in the CCM, however an experimental verification of these predictions would be of further interest.

Next, the role of $Q$ value is investigated in figure 6 for the 1n- and 2n-halo cluster configurations in the ground state of various parent nuclei. The $Q$ values are related to the mass difference between the parent nucleus and the cluster and core nuclei, which in turn is related to the corresponding binding energy differences. We find in figure 6 that all the nuclei, except $^{26}\text{F}$ and $^{31}\text{Ne}$, considered here are stable ($Q < 0$) against 1n- and 2n-cluster formations. On the other hand, the $Q$ value is positive against $1n^{25}\text{F}$ and $1n^{30}\text{Ne}$ configurations of $^{26}\text{F}$ and $^{27}\text{Ne}$, respectively. In other words, both $^{26}\text{F}$ and $^{31}\text{Ne}$ nuclei seem relatively unstable against one-neutron clusterization.

Finally, the possible role of a variety of nuclear proximity potentials is investigated in order to check the anomaly associated with neutron-halo structures of the above studied $^{22}\text{O}$, $^{26}\text{F}$ and

Figure 5. The fragmentation potential for (a) $^{22}\text{O}$, (b) $^{26}\text{F}$ and (c) $^{27}\text{F}$ nuclei, taking the two fragments as spheres, $\beta_2$ alone, and ($\beta_2-\beta_4$) deformations at $\ell = 0$. 
Figure 6. The calculated $Q$-values against 1n- and 2n-cluster configurations from various parent nuclei in the ground state.

Figure 7. Same as for figure 5, but for the use of different proximity potentials, Prox 77, Prox 88, and Prox 00, using $(\beta_2 \cdot \beta_4)$ deformations and compact orientations.

$^{27}$F nuclei for use of the higher multipole deformations up to $\beta_4$. It is important to note that so far only the proximity potential Prox 77 of Blocki et al [31] is used in the framework of CCM, so also in all the above discussion. Figure 7 presents the comparison of our PES using nuclear proximity potentials Prox 77, Prox 88 and Prox 00, having different dependences on isospin and asymmetry parameters, for the case of $\beta_2 \cdot \beta_4$ deformed choice of fragments. Although the magnitudes of fragmentation potential are quite different for Prox 77 and Prox 88 potentials, their PES behavior is similar and gives 2n-halo configuration for $^{22}$O, $^{26}$F and 1n-halo for $^{27}$F nucleus. Prox 00, on the other hand, suggests 1n-halo for $^{22}$O and $^{26}$F, without any change in the structure of $^{27}$F. As already noted above, proximity potentials Prox 77 and Prox 88 use the same universal function and its parameters, except the values of two coefficients of surface energy term, whereas Prox 00 uses an entirely different formulation, and hence these nuclear
interactions are expected to behave differently. As a result Prox 00 gives a relatively smooth potential surface for the cluster masses $A_2 \leq 4$ in comparison to the behavior presented by Prox 77 and Prox 88 potentials. Therefore, one may conclude that the choice of proximity potential, together with deformation and orientation effects, play a significant role in deciding the halo structure of a variety of neutron drip-line nuclei. Hence, the information contained in the present work is of extreme relevance in reference to nuclear structure and related aspects associated with $^{17}$B, $^{22}$C, $^{22}$O, $^{26}$F and $^{27}$F halo nuclei.

4. Summary

Summarizing, the role of deformations in the ground state clusterization of some 24 light neutron-rich nuclei is investigated by using the CCM in terms of the PES calculated as a sum of binding energies, Coulomb repulsion, nuclear proximity attraction and the centrifugal potential for all the possible cluster + core configurations of a nucleus. Besides deformations, the comparison of hot (compact) and cold (non-compact) orientations is also analyzed. Interestingly, for a few nuclei, we find that, with the inclusion of deformations ($\beta_2$ alone or $\beta_2-\beta_4$), the minimum in the fragmentation potential shifts to another cluster. The cases of both 1n- and 2n-halo structures are identified on the basis of CCM predictions, which show some differences with the neutron separation energy hypothesis. The possible role of deformations and related orientations is analyzed in reference to new magic numbers at $N = 6, 12, 14, 16,$ and $18$ and $Z = 6$, which are shown to be associated with the shell effects of the core nucleus. Furthermore, the cluster structure of n-halo nuclei is found to be sensitive to the use of different forms of proximity interaction. Thus, the present study points out the significance of both the deformations and the proximity interaction, specifically for $^{17}$B, $^{22}$C, $^{22}$O, $^{26}$F and $^{27}$F n-halo nuclei.

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