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Present and future of the sub‐barrier fusion reactions
Role of energy dependent interaction potential in sub-barrier fusion of $^{28}\text{Si} + ^{90}\text{Zr}$ system

Manjeet Singh Gautam† and Manoj K. Sharma

†School of Physics and Material Science, Thapar University, Patiala (Punjab)-147004, India
†gautammanjeet@gmail.com

Abstract: We have analyzed the importance of the inelastic surface vibrations of colliding nuclei in the sub-barrier fusion enhancement of $^{28}\text{Si} + ^{90}\text{Zr}$ system by using the energy dependent Woods-Saxon potential model (EDWSP model) in conjunction with one dimensional Wong formula and the coupled channel formulation using the code CCFULL. The multi-phonon vibrational states of colliding nuclei seem to impart significant contribution. The coupling between relative motion of reactants and these relevant channels in turn produce anomalously large sub-barrier fusion enhancement over the expectations of one dimensional barrier penetration model. Furthermore, the effects of coupling to inelastic surface excitations are imitated due to energy dependence in the Woods-Saxon potential. In EDWSP model calculations, a wide range of diffuseness parameter much larger than the elastic scattering predictions is needed to account the observed fusion enhancement in the close vicinity of Coulomb barrier.

Keywords: Heavy-ion near-barrier fusion, Woods-Saxon potential, Coupled channel equations, Diffuseness anomaly.

INTRODUCTION

In heavy ion reactions, the availability of radioactive beam strengthens the systematic study of complex phenomenon of multidimensional quantum tunneling wherein the projectile and target with many degrees of freedom penetrate through the classical forbidden energy regions. Fusion reactions, which can be used as spectroscopic tool to explore nature of nuclear interactions and nuclear structure of participating nuclei, have received attention in the recent years because the energy dependence of the fusion cross-sections depend strongly on the internal structure of the reacting nuclei such as nuclear shape deformation, vibrations of nuclear surface, successive rotation of nuclei during collision, neck formation and nucleon (multi-nucleon) transfer mechanism etc. The coupling between relative motion and internal structure degrees of freedom causes the splitting of the uncoupled Coulomb barrier into a distribution of barriers. This barrier distribution is the direct manifestation of the fusion enhancement at energies below the energy of uncoupled Coulomb barrier [1]. Theoretically, the simplest way to describe the fusion mechanism is the barrier penetration model (BPM), wherein the collision partners are assumed to penetrate through the fusion barrier and form a composite nucleus. However, an anomalously large enhancement in the fusion cross-section over the predictions of one dimensional barrier penetration model at sub-barrier energies has been observed during last two decades [1].

Indeed, the couplings of relative motion of colliding pairs and internal structure degrees of freedom produce substantially large enhancement in the fusion cross-section at sub-barrier energies. Theoretically, it is very difficult to include all intrinsic degrees of freedom simultaneously, but to identify those degrees of freedom which have strong influence on the fusion mechanism and consequently to understand the puzzling dynamics of the fusion process has been a matter of considerable interest in recent past. The relation between sub-barrier fusion...
enhancement and intrinsic degrees of freedom such as permanent shape deformation, low lying surface vibration of fusing nuclei have been well described by the various theoretical formulations [1]. However, the interplay of neutron transfer channels have not been fully understood because neutron transfer mechanism is insensitive to the Couomb barrier and such transfer processes generally occur at large internuclear separation. As a result, the fusion reactions have become the most studied processes to explore the importance of structural as well as dynamical effects associated in the compound nuclear reactions. In the present study, the fusion dynamics of $^{28}_{14}$Si + $^{90}_{40}$Zr system [2] is analyzed within the context of EDWSP model [3] and coupled channel approach. Role of inelastic surface excitations of colliding pairs are properly entertained by using the coupled channel calculations. It is worth noting that the predictions of the EDWSP model and coupled channel model closely resemble and hence adequately describe the sub-barrier fusion enhancement observed in the chosen reaction. The brief description of methodology is given in section 2. The results are discussed in section 3 while the conclusions drawn are presented in section 4.

**THEORETICAL FORMULATION**

**One dimensional Wong formula**

The partial wave expansion for reaction cross-section leads to the following expression for the fusion cross-section

$$\sigma_f = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell + 1) T_{\ell}^F$$  \hspace{1cm} (1)

In order to obtain a simple expression for the fusion probability ($T_{\ell}^F$), one usually uses the Hill and Wheeler approximation wherein the effective potential near the barrier radius is approximated by a parabola [4].

$$T_{\ell}^{\text{Hill-Wheeler}} = \frac{1}{1 + \exp \left( \frac{2\pi}{\hbar \omega} (V_r - E) \right)}$$  \hspace{1cm} (2)

The above Hill-Wheeler expression is exact for a parabolic barrier and is approximate for potential barriers in heavy-ion collisions. Wong further simplifies this parabolic approximation by assuming that the infinite number of partial waves contribute to the fusion process and obtain the following expression for evaluating the fusion cross-section [5].

$$\sigma_f = \frac{\hbar \omega R_a^2}{2E} \ell n \left[ 1 + \exp \left( \frac{2\pi}{\hbar \omega} (E - V_r) \right) \right]$$  \hspace{1cm} (3)

**Energy dependent Woods-Saxon Potential model (EDWSP model)**

In the present analysis, the EDWSP model in conjunction with one dimensional Wong formula is used for theoretical calculations of sub-barrier fusion excitation function [3]. The form of static Woods-Saxon potential is defined as

$$V_s(r) = \frac{-V_0}{1 + \exp \left( \frac{r - R_0}{a} \right)}$$  \hspace{1cm} (4)

with $R_0 = r_0 \left( A_1^{\frac{1}{3}} + A_2^{\frac{1}{3}} \right)$. The quantities ‘$V_0$’ and ‘$a$’ are respectively the strength and the diffuseness parameter of the nuclear potential. In EDWSP model, the depth of real part of Woods-Saxon Potential is defined as
where $I_p = \left( \frac{N_p - Z_p}{A_p} \right)$ and $I_t = \left( \frac{N_t - Z_t}{A_t} \right)$ are the isospin asymmetry of projectile and target nuclei respectively. The above parameterization of the potential depth is based upon the reproduction the fusion excitation function data of various projectile-target combinations ranging from $Z_pZ_t = 84$ to $Z_pZ_t = 1640 [3]$. In literature, significantly larger values of diffuseness parameter ranging from $a = 0.75 \text{ fm}$ to $a = 1.5 \text{ fm}$ are required for reproduction of fusion excitation function data [1]. Owing to importance of diffuseness parameter, the energy dependence in the Woods-Saxon potential is taken through its diffuseness parameter and hence is defined as

$$a(E) = 0.85 \left[ 1 + \frac{r_0}{13.75(\frac{A_p}{A_p} + \frac{A_t}{A_t})^2} \left( 1 + \exp \left( \frac{E - 0.96}{0.03} \right) \right) \right] \text{ fm}$$

(6)

In the EDWSP model calculations, this expression provides a wide range of values of diffuseness depending upon the value of $r_0$ and bombarding energy of colliding pairs. The free parameter $r_0$ is varied to reproduce the fusion excitation function data of system under consideration.

### Coupled Channel Model

In this section, the details of coupled channel approach have been discussed. In the heavy ion fusion reactions, the couplings of relative motion with intrinsic degrees of freedom of the colliding systems are playing very important role in the enhancement of sub-barrier fusion cross-section in the close vicinity of Coulomb barrier. Theoretically, the standard way to address various channel coupling effects is to deal with the coupled channel equations [6]. Therefore, the set of coupled channel equation can be written as

$$\left[ \frac{-\hbar^2}{2\mu} \frac{d^2}{dr^2} + \frac{J(J+1)\hbar^2}{2\mu r^2} + V_N(r) + \frac{Z_pZ_t e^2}{r} + E_a - E_{cm} \right] \psi_n(r) + \sum_m V_{nm}(r) \psi_m(r) = 0$$

(7)

here, $r$ is the radial coordinate for the relative motion between fusing nuclei. $\mu$ is defined as the reduced mass of the fusing nuclei. The quantities $E_{cm}$ and $E_a$ represent the bombarding energy in the centre of mass frame and the excitation energy of the $n^{th}$ channel respectively. The $V_{nm}$ is the matrix elements of the coupling Hamiltonian, which in the collective model consists of Coulomb and nuclear components. For the coupled channel calculations e the code CCFULL [6] has been used. In this code, the coupled channel equations are solved numerically by imposing the certain basic approximations such as rotating frame approximation or no Coriolis approximation, ingoing wave boundary conditions. By including all the relevant channels, the fusion cross-section can be written as

$$\sigma_f(E) = \sum_j \sigma_j(E) = \frac{\pi}{k_0^2} \sum_j (2J + 1) P_j(E)$$

(8)

where, $P_j(E)$ is the total transmission coefficient corresponding to the angular momentum $J$. 
RESULTS AND DISCUSSION

The energy dependent Woods-Saxon potential proposed in the earlier work [3] is used to explain the various aspects related to fusion dynamics of nuclear systems in the close vicinity of Coulomb barrier. The inclusion of energy dependence in real part nuclear potential in such a way that it becomes more attractive at sub-barrier energies is another alternative way to govern similar kind of physical effects as induced due to internal degrees of freedom of reactants [3]. Such kind of energy dependence in the nucleus-nucleus potential lowers the effective fusion barrier between fusing nuclei and hence ultimately predicts relatively larger sub-barrier fusion cross-sections with respect to energy independent one dimensional barrier penetration model as evident from the present work (EDWSP model).

In order to understand the rich interplay of inelastic surface vibrational states, the fusion dynamics of $^{28}_{14}\text{Si} + ^{90}_{40}\text{Zr}$ system is analyzed in the present work. The influence of inelastic surface vibrations of collision partners is observed to be dominant [2] and we intend to focus on this issue. The $^{28}_{14}\text{Si}$ nucleus is non magic nucleus while the $^{90}_{40}\text{Zr}$ nucleus is doubly magic and both collision partners suggest the low lying surface vibrations. For this system the diffuseness parameter $'a'$ varies from $a = 0.97$ fm to $a = 0.85$ fm in the energy range from $E_{c.m.} = 60$ MeV to $E_{c.m.} = 95$ MeV. The value of depth parameter ($V_0$) comes out to be $84.23$ MeV while the range parameter $'r_0'$ is kept fixed at $1.080$ fm. Very recently, an energy dependent Woods-Saxon potential model (EDWSP model) has been successfully used to explore the fusion dynamics of various heavy ion systems [3]. The present work has been extended for analysis of the fusion of $^{28}_{14}\text{Si} + ^{90}_{40}\text{Zr}$ system within the framework of EDWSP model and the coupled channel formulation.

![Fig.1](image-url)

Fig.1. Fusion excitation functions of $^{28}_{14}\text{Si} + ^{90}_{40}\text{Zr}$ system obtained by using the EDWSP model [3] in conjunction with one dimensional Wong formula [5] and coupled channel code CCFULL [6], compared with the available experimental data [2].

In Fig.1, we compare the fusion excitation functions of $^{28}_{14}\text{Si} + ^{90}_{40}\text{Zr}$ system obtained by using the energy dependent Woods-Saxon potential model (EDWSP model) in conjunction with one dimensional Wong formula and the coupled channel calculations performed by using the code CCFULL along with the corresponding data. Various coupled channel calculations predicted that the couplings to inelastic surface vibrational states of both colliding nuclei are necessarily required to reproduce the sub-barrier fusion excitation data. For no coupling calculations...
wherein the colliding nuclei are considered as inert, one observe substantially smaller fusion contribution. The inclusion of one phonon $2^\nu$ and $3^\nu$ vibrational states of target significantly improves the results but still fails to provide the complete description of experimental data at below barrier regions. This demands the inclusion of higher order multi-phonon vibrational coupling that must be incorporated in the coupled channel calculations to reconcile the experimental data. The inclusion of one phonon $2^\nu$ state in projectile, one phonon $2^\nu$ state and two phonon $3^\nu$ vibrational states in target bring the required order of magnitude of sub-barrier fusion enhancement. It is quite interesting to note that the EDWSP model along with one dimensionl Wong formula adequately addresses the sub-barrier fusion enhancement of $^{28}\text{Si} + ^{90}\text{Zr}$ system. This clearly suggest the fact that the energy dependence in Woods-Saxon potential mocks up various channel coupling effects as evidently depicted in Fig.1. In EDWSP model calculations, the energy dependence in Woods-Saxon potential produces a spectrum of energy dependent barriers of varying heights. The passage through the barrier whose height is smaller than that of Coulomb barrier is more probable and hence accurately explains the fusion dynamics of $^{28}\text{Si} + ^{90}\text{Zr}$ system.

CONCLUSIONS

In summary, the predictions of the EDWSP model and the coupled channel formulation closely resemble and adequately explain the sub-barrier fusion enhancement of $^{14}\text{Si} + ^{40}\text{Zr}$ system. This unambiguously mirrors that EDWSP model introduces barrier modification effects in somewhat similar way to that of various channel coupling effects and hence the energy dependence in the Woods-Saxon potential mimics various kinds of static and dynamical physical effects in the sub-barrier fusion dynamics.

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